The valuation pairing on an upper cluster algebra

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It is known that many (upper) cluster algebras are not unique factorization domains. In order to study their local factorization properties, we introduce the valuation pairing on any upper cluster algebra U. To each pair (a, u) consisting of a cluster variable a and a non zero element u of U, it associates the largest integer v such that that $\frac{u}{a^v}$ still belongs to U. Using the valuation pairing we prove that any full rank geometric upper cluster algebra has the following local unique factorization property: For each seed t of U, any non-zero element $u \in U$ can be uniquely factored as $u = ml$, where m is a cluster monomial in the seed t and l is an element in U not divisible by any cluster variable in t. We have many applications to d-vectors, F-polynomials, factoriality of upper cluster algebras and combinatorics of cluster Poisson variables. In this talk, we focus on the application to d-vectors. We will show how to express d-vectors using the valuation pairing. This is a report on joint work with Bernhard Keller and Fan Qin.